Different approach for evaluating dissipation in macroscopic quantum tunneling

A. Agresti,[1] M. Barilli,[2] A. Ranfagni,[2,3] R. Ruggeri,[3] and C. Susini[3]
[1]*Dipartimento di Fisica dell'Università di Firenze, Sezione di Fisica Superiore, and Istituto Nazionale di Fisica della Materia, Unità di Ricerca di Firenze, Firenze, Italy*
[2]*Scuola di Specializzazione in Ottica dell'Università di Firenze, Firenze, Italy*
[3]*Istituto di Fisica Applicata "Nello Carrara," Consiglio Nazionale delle Ricerche, Via Panciatichi 64, 50127 Firenze, Italy*
(Received 20 February 2002; revised manuscript received 26 March 2002; published 25 June 2002)

The problem of evaluating dissipative effects in macroscopic quantum tunneling is re-examined for the case of Josephson junctions, with the adoption of an alternative way with respect to several previously proposed and, in some cases, contradictory approaches. The system, which consists of a junction coupled to a transmission line, is analyzed both analytically and numerically. A test of the theoretical model, as compared to the experimental results available, is performed in accordance with a criterion based on a shortening of the traversal time.

DOI: 10.1103/PhysRevE.65.066616 PACS number(s): 74.50.+r, 03.65.Sq, 73.40.Gk

I. INTRODUCTION

In several aspects, the motion of a particle undergoing quantum tunneling—one of the more nonclassical predictions of quantum mechanics—has long been an open problem. Today, the focus of much tunneling research is on determining the degree to which this quantum concept can be extended to the macroscopic world [1]. This leads to a need to include, within a quantum framework, the effect of the dissipation, which is more or less always present in macroscopic systems. Within this context, the Josephson effect is one of the most suitable for observing phenomena such as macroscopic quantum tunneling (MQT), macroscopic quantum coherence (MQC), and energy level quantization (ELQ) [2].

The evaluation of dissipative effects in MQT and MQC is a formidable task, and since the 1980s many efforts have been devoted to dealing with it. A relatively recent work on this topic [3] reopened a question already debated [4] and only apparently fully understood [5]. In fact, it might be argued that dissipation always suppresses tunneling because it induces an additional positive Euclidean action. However, this is not necessarily true since, in some approaches, the tunneling effect depends considerably on the choice of counterterms. Again, in Ref. [3] the possibility that dissipation enhances the tunneling rate is demonstrated, as opposed to the generally accepted tendency [6,7].

Subsequently, however, this problem was redimensioned, demonstrating that whether or not this counterterm should be included essentially depends on the problem concerned [8]. In other words, depending on the model used, the inclusion of the counterterm may be compulsory. Since an attempt is being made to avoid microscopic analysis, due to its intrinsic complexity, the counterterm is found to be very natural within the context of more phenomenological approaches [6].

However, in accepting this point of view it is possible to follow different approaches in which this problem does not arise. For example, we can adopt a procedure that is mainly based on an evaluation of the energy lost by the system while tunneling. In this way, we can obtain results that are comparable to those obtained by using different, more sophisticated (i.e., requiring functional integrations) procedures. A result of this type was anticipated in Ref. [9].

The present work is organized as follows. First, we present a short review of the main contributions for evaluating the dissipative effects, mainly for the case of a Josephson junction. In Sec. II, using a different procedure, we reconsider the problem of a Josephson junction coupled to a transmission line, which controls the dissipative effects. Last, in Sec. III, on the basis of experimental results available up until now a test of the model is performed according to a new criterion.

The issues of the principal approaches to the problem are summarized in Table I. The results in point (a)—where η is the friction coefficient, ω_c is the cutoff frequency of the phonon-bath, τ_B is the bounce duration, and x_B is its amplitude—are direct consequences of the Feynman approach [10] as derived by Sethna [11], and by Bruinsma and Per Bak [12], the latter having been obtained in the short-time limit, which is the opposite of the one of interest to us. Within both limits, $\omega_c\tau_B \ll 1$ and $\omega_c\tau_B \gg 1$, these give a negative variation of the action, which would produce an enhancement of the decay rate. However, as previously anticipated, it is commonly believed that a friction coefficient in the motion (even if classically forbidden) must produce a suppression of the tunneling. As previously mentioned, this apparent contradiction was resolved by introducing a counterterm that neutralizes the variation in the local potential in the Lagrangian of the tunneling system coupled to the phonon bath. This produced the results in point (b), results which are at least qualitatively confirmed by the other approaches [5]. The approaches in points (c) and (d), while producing more suitable results for applications, always rest on the same assumption of renormalizing the local potential by selecting an appropriate counterterm. The variation in the numerical coefficient multiplying ηx_B^2 in point (c) is due to different values of the ratio τ_B/τ_k of the bounce duration τ_B to the duration τ_k of a single kink [13]. The result in point (d) is considered to be the most appropriate for interpreting a variety of systems [6].

The approach in point (e) is said to be phenomenological,

TABLE I. Different approaches for evaluating the dissipative action-variation for the bounce trajectory of a Josephson junction.

Variation of the action ΔS		References
(a)Path-integrals	$-(\eta/4\pi)x_B^2[\omega_c^2\tau_B^2-O(\omega_c^3\tau_B^3)],\ \omega_c\tau_B\ll 1$	[11,12]
	$-(\eta/\pi)x_B^2[\omega_c\tau_B-\ln(\omega_c\tau_B)-\text{const}],\ \omega_c\tau_B\gg 1$	[10,11]
(b)Path-integrals with	$(\eta/\pi)x_B^2[\omega_c\tau_B-O(\omega_c^2\tau_B^2)],\ \omega_c\tau_B\ll 1$	[5]
renormalized potential	$(\eta/\pi)\ln\lvert\omega_c\tau_B\rvert x_B^2+\text{const},\ \omega_c\tau_B\gg 1$	[5]
(c)Alternative formula for (b)	$(\eta/\pi)[3/2+\ln(\tau_B/\tau_k)]x_B^2\simeq(0.48-0.83)\,\eta x_B^2$	[13]
$\omega_c\tau_B\gg 1$, with $\tau_B/\tau_k=1-3$		
(d) Caldeira Leggett analysis	$(\eta/2)\int_{-\infty}^{\infty}\lvert\omega\rvert\lvert\xi(\omega)\rvert^2 d\omega\simeq 0.465\,\eta x_B^2$	[6]
with counterterm		
(e)Phenomenological analysis	$\eta\int_{-\infty}^{\infty}d\tau\int_{-\infty}^{\tau}d\tau'[\dot{x}(\tau')]^2\simeq 0.87\,\eta x_B^2$	[9,15]
(f) Distributed load	$\int_{-\infty}^{\infty}H(\omega)\lvert\xi(\omega)\rvert^2 d\omega,\ H(\omega)=\eta\lvert\omega\rvert\tanh(\lvert\omega\rvert\tau_0)$	[16,18]
(open transmission line)		

since it is derived by simply evaluating the energy lost during tunneling. In fact, no energy is lost in the tunneling process if the particle is isolated from its environment, whereas coupling to the environment means that the particle loses energy while tunneling [14]. This can be seen either as producing a loss of energy [9] or as an increase in the potential of the barrier, which is given by $\eta\int_{-\infty}^{\tau}|\dot{x}|\dot{x}\,d\tau'$ [15]. Then, by integrating again over time, we obtain the result in point (e), which is comparable with the ones in points (c) and (d). It is noteworthy that the range of the numerical factor of the limits in point (c) has values that are practically coincident with the numerical factors of the limits in points (d) and (e). Therefore, all of these treatments are really comparable.

Last, a completely different treatment is the one in point (f). Developed by Chakravarty and Schmid [16], it was almost ignored in subsequent literature [17], but recently reexamined [18]. The peculiarity of this approach consists of the incorporation of a distributed circuit model—a transmission line that determines the dissipative effects—within the bounce formalism. The results obtained therein confirm those of different approaches, but avoid any *ad hoc* assumptions. The approaches in points (e) and (f) lead to very similar results, as can be seen below, devoting our analysis to a better understanding of this similarity.

II. AN ALTERNATIVE WAY OF DETERMINING DISSIPATIVE EFFECTS

First, let us consider the circuit arrangement of Fig. 1(a), where a Josephson junction is coupled to an open transmission line of indefinite length z. Let us denote the inductance and the capacitance per unity length by l and c, respectively. Thus, the characteristic impedance of the line (assuming the electrical loss to be negligible) is $Z_0=(l/c)^{1/2}$, and the wave velocity is $v=(lc)^{-1/2}$. With z' being the spatial coordinate and τ being the imaginary time, the Euclidean Lagrangian of the line can be expressed as

$$\mathcal{L}(z,\tau)=\int_0^z[\tfrac{1}{2}lI^2(z',\tau)+\tfrac{1}{2}cV^2(z',\tau)]dz', \tag{1}$$

where the current $I(z',\tau)$ and the voltage $V(z',\tau)$ are related by $V(z',\tau)=Z_0I(z',\tau)$. By substituting in Eq. (1) and changing the variable according to $dz'=v\,d\tau'$, we obtain

$$\mathcal{L}(z,\tau)=\int_0^z cV^2(z',\tau)dz' = \int_0^\tau cV^2(\tau')v\,d\tau' = \int_0^\tau Z_0^{-1}V^2(\tau')d\tau', \tag{2}$$

and the relative action (ΔS) will be obtained by a further integration in time. The action of the junction alone will be denoted by S_J, and is given by

$$S_J=\int d\tau\mathcal{L}[\phi(\tau)], \tag{3}$$

where $\varphi(\tau)$ is the Cooper-pair phase-difference across the junction. By assuming that the internal dissipation of the junction is negligible, $\varphi(\tau)$ is given by [5]

$$\varphi(\tau)=\varphi_B\text{sech}^2\left(\frac{\Omega\tau}{2}\right), \tag{4}$$

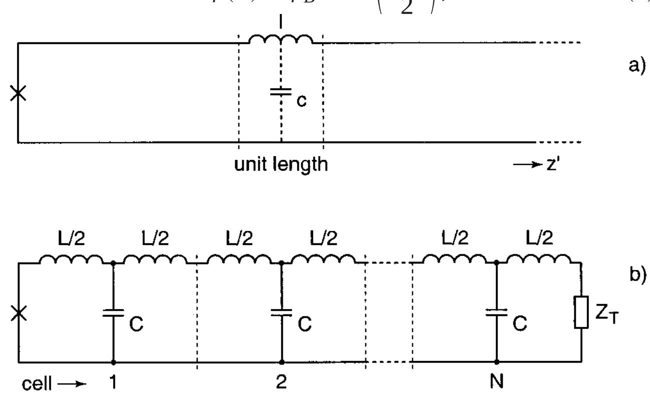

FIG. 1. Josephson junction coupled to an open transmission line (a), where l and c represent the inductance and the capacitance per unit length. In (b), the junction is coupled to an artificial line consisting of N cells of T type, terminated with the impedance Z_T.

where φ_B is the bounce amplitude, and Ω is the angular frequency of small oscillations inside the potential well. Dissipative effects are attributed only to the transmission line, but the bounce trajectory (4) is assumed to be unchanged [19]. Since the line and the junction are coupled at $z'=0$, the second Josephson equation must have $V(z'=0,\tau) =(\Phi_0/2\pi)\dot{\varphi}(\tau)$, where Φ_0 is the flux quantum. Therefore, Eq. (2) simply becomes

$$\mathcal{L}(\tau)=\left(\frac{\Phi_0}{2\pi}\right)^2\frac{1}{Z_0}\int_0^{\tau}[\dot{\varphi}(\tau')]^2 d\tau', \tag{5}$$

a result that strictly resembles the first integral (in $d\tau'$) in point (e) by identifying $(\Phi_0/2\pi)^2 Z_0^{-1}$ with η, and $\dot{\varphi}(\tau)$ with $\dot{x}(\tau)$. There is, however, some significant difference between the two results since in point (e) the variable τ runs from $-\infty$ to $+\infty$, whereas in the present case $\tau=z/v$ runs from 0 to $+\infty$ (or from $-\infty$ to 0). Analogously, and coherently with this constraint, the integration in Eq. (3) will also be limited in the same time interval for a correct consideration of the relative importance. Therefore, it seems that the action integral for the line should be evaluated as

$$\Delta S=\left(\frac{\Phi_0}{2\pi}\right)^2\frac{1}{Z_0}\int_{-\infty}^{0}d\tau\int_{-\infty}^{\tau}[\dot{\varphi}(\tau')]^2 d\tau', \tag{6}$$

which gives a result of just half of that in point (e), namely, $0.436\eta\varphi_B^2$. Note that once the substitution $\dot{x}d\tau=dx$ is made, Eq. (6), or, better, its equivalent expression $\Delta S =\eta\int_{-\infty}^{0}d\tau\int_{-\infty}^{\tau}d\tau'[\dot{x}(\tau')]^2$, becomes $\Delta S=\eta\int_0^x dx'/\dot{x}(x')\int_0^{x'}\dot{x}(x'')dx''$, which is exactly equal to Eq. (11) in Ref. [9], x being the spatial coordinate in the barrier. In our case, the spatial coordinate is, from the beginning, z', which runs from zero to an indefinite length $z\to\infty$ of the transmission line. In both cases, however, the action integral ΔS can be expressed by means of $\dot{x}(\tau)\to\dot{\varphi}(\tau)$, and by considering only a half-bounce trajectory, since in real tunneling processes only a half bounce is actually traveled by the particle. In terms of probability, which is the square of the absolute value of amplitude, the calculation requires consideration of the complete bounce [20]. A consideration of the complete bounce trajectory within our framework, i.e., from Eq. (1) to Eq. (6), naturally rests on the assumption of a symmetric transmission line, as in Ref. [16], that is, with z' running from $-z$ to $+z$ (see Fig. 2). Consequently, the time variable runs from $-\infty$ to $+\infty$, and ΔS is twice that of Eq. (6), namely, $0.872\eta\varphi_B^2$. Adopting a symmetric line is rather unnatural, and a suitable geometry for measurements consists of a junction coupled to a single transmission line. Considering a pair of parallel transmission lines merely reduces the situation to the case of only one, with a characteristic impedance $Z_0/2$ [18].

In order to test the worth of the result expressed by Eq. (6), we have reexamined the problem on the basis of an alternative approach. We followed a procedure that is halfway between the one adopted by Chakravarty and Schmid,

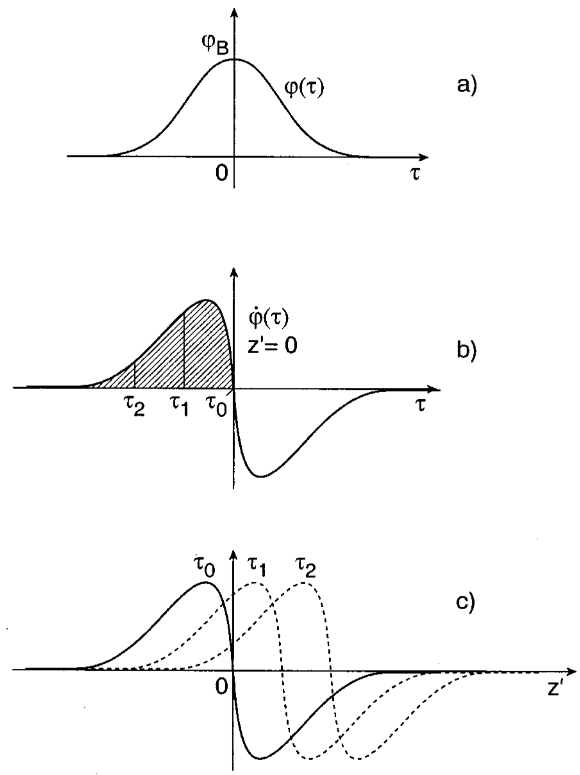

FIG. 2. Sketch of the voltage pulse $V(\tau,z')\propto\dot{\varphi}(\tau)$ traveling along a transmission line with characteristic impedance $Z_0=V/I$. In (a), we have the bounce trajectory $\varphi(\tau)$ centered at $\tau=0$. In (b), the pulse is given as a function of time at $z'=0$. In (c), the pulse, which travels in the direction of increasing z', is represented as a function of the spatial coordinate z' at different instants τ_0,τ_1,τ_2. For a given coordinate (eg., $z'=0$), the pulse values subsequent to $\tau=0$ correspond to the points marked in (b), which are situated in the domain of negative times. Analogously, for a pulse traveling in the opposite direction, it is necessary to consider the domain of positive times.

point (f) in Table I, and that of Widom and Clark [21]. As previously anticipated, the advantages of such a procedure lie in demonstrating how to incorporate a distributed, or lumped, circuit model in bounce formalism in order to calculate the tunneling rate from the zero-voltage state. This way we avoid any *ad hoc* assumptions, as well as delicate boundary conditions that are inherent in the approach of point (f) [18].

For this purpose, we have considered the circuit arrangement of Fig. 1(b) with a Josephson junction coupled to an artificial transmission line consisting of a number N of cells with inductance L and capacitance C. This arrangement can either be terminated with an impedance Z_T equal to the characteristic one $Z_0=\sqrt{L/C}$, or opened ($Z_T=\infty$), similarly to the previously adopted case of Fig. 1(a). Analogously, the wave velocity is $v=(LC)^{-1/2}$ and the delay is given by $\tau_0 =N\sqrt{LC}$. These relations hold true if the angular frequency ω is sufficiently lower than the cutoff one at $\omega_c=2/\sqrt{LC}$.

Analogously to Eq. (1), the Euclidean Lagrangian of the line was expressed as [22]

TABLE II. The currents I_i and the voltages V_i of the Lagrangian (7) can be determined as the imaginary frequency $s=i\omega$, and for a given excitation V_0, by solving a system of N algebraic equations whose coefficients are listed below. For open lines, the last coefficient of V_N becomes zero.

V_0	I_1	V_1	I_2	V_2	I_3	V_3	...	I_{N-1}	V_{N-1}	I_N	V_N
1	$sL/2$	1	0	0	0	0	...	...	...	...	...
0	1	$-sC$	-1	0	0	0	...	...	...	...	...
0	0	1	$-sL$	-1	0	0	...	...	...	...	...
0	0	0	1	$-sC$	-1	0	...	...	...	...	...
0	0	0	0	1	$-sL$	-1	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...
0	...	...	...	...	...	...	...	1	$-sC$	-1	0
0	...	...	...	...	...	...	...	0	1	$-sL/2$	-1
0	...	...	...	...	...	...	...	0	0	1	$-\sqrt{C/L}$

$$\mathcal{L}(N,\tau)=\frac{1}{4}LI_1^2(\tau)+\frac{1}{4}LI_N^2(\tau)+\sum_{i=2}^{N-1}\frac{1}{2}LI_i^2(\tau)+\sum_{i=1}^{N-1}\frac{1}{2}CV_i^2(\tau)+\tau_0V_N(\tau)I_N(\tau), \tag{7}$$

where I_i and V_i are, respectively, the current and the voltage of the different cells. These can be obtained, for a given excitation $V_0(\tau)$, by solving a system of differential equations. However, since our V_0 is obtained in an imaginary-time variable (bounce formalism), it is convenient to solve our circuit problem in imaginary frequencies; that is, according to a Laplace-transform analysis that works in a temporal semispace and fits well into our problem. This simplifies the procedure enormously, since the problem is reduced to solving a system of algebraic equations (see Table II). In this way, the Lagrangian of Eq. (7) becomes a function of frequency ω, that is, $\mathcal{L}(i\omega,\tau_0)$, and the action of the line can be evaluated as [16,18]

$$\Delta S(\Omega,\tau_0)=\int_{-\infty}^{\infty}d\omega F(\omega,\tau_0)|\xi(\omega)|^2, \tag{8}$$

where $F(\omega,\tau_0)=\omega^2\mathcal{L}(i\omega,\tau_0)$, and $\xi(\omega)$ is the Fourier transform of the trajectory $\varphi(\tau)$ of Eq. (4), defined as in Ref. [5], namely,

$$\xi(\omega)=\varphi_B\frac{2\sqrt{2}}{\Omega\sqrt{\pi}}\left(\frac{\pi\omega}{\Omega}\right)\mathrm{csech}\left(\frac{\pi\omega}{\Omega}\right). \tag{9}$$

By taking into account that $|V(\omega)|=(\Phi_0/2\pi)\omega\xi(\omega)$, and that $[(\Phi_0)/2\pi]^2=\eta Z_0$, Eq. (8) becomes

$$\Delta S(\Omega,\tau_0)=\left(\frac{2\sqrt{2}}{\Omega\sqrt{\pi}}\right)^2\eta\varphi_B^2Z_0\int_{-\infty}^{\infty}d\omega\,\omega^2\mathcal{L}(i\omega,\tau_0)\times\left(\frac{\pi\omega}{\Omega}\right)^2\mathrm{csech}^2\left(\frac{\pi\omega}{\Omega}\right). \tag{10}$$

Since the trajectory is a function of Ω, which determines its duration, ΔS also turns out to be a function of Ω, in addition to τ_0, which is a measure of the virtual length of the artificial line.

Notwithstanding the relative simplicity of this procedure with respect to evaluating functional integrations, the results could easily be obtained simply by means of numerical analysis. Several computations have been performed, and some results are shown in the form of continuous lines in Figs. 3 and 4, as functions of Ω and in units of $\eta\varphi_B^2$. They refer to an artificial line composed of N cells with $L=0.1$ and $C=0.2$ so that the cutoff frequency ω_c is above 14, and N ranges from 2 to 20. Therefore, the delay τ_0 varies in the 0.28–2.8 range. The curves in Fig. 3 were obtained with the artificial line terminated with the characteristic impedance $Z_0=0.707$, while the curves in Fig. 4 refer to the open artificial line, $Z_T=\infty$. In both figures, our results are superimposed on a family of curves (dashed lines) that represents the results of the model in point (f) of Table I for several values of the same parameter τ_0. In the case of Fig. 3, they are properly divided by 2 since the formula in point (f) refers to two open parallel transmission lines [16]. We note that while the two approaches tend to be comparable within the limit of large values of Ω, there is a disagreement of roughly a factor of 2 within the opposite limit of small values of Ω. This means that, for a long pulse duration (or short line), our results tend to be twice those of Ref. [16]. The results of Fig. 4 are superimposed on a family of curves, as given directly by the formula in point (f) (without dividing by two). In this case we note that while in the large Ω limit, our results, situated at best within $\sim$0.4–0.6, $\eta\varphi_B^2$, are sensibly lower than the limiting value of the other model, in the small Ω (capacitive) limit they tend to be in rather good agreement. Under these circumstances, it seems that none of the reported models in Table I has to be considered as definitive; instead, the situation seems to be a little more complicated depending also on the region (Ω,τ_0) of the parameters that we are considering.

For a better comparison, it is instructive to report in the same diagram (see Fig. 5) the numerical results already shown in Figs. 3 and 4. What clearly emerges is that, within the limit of high-frequency Ω, and for a sufficient number N

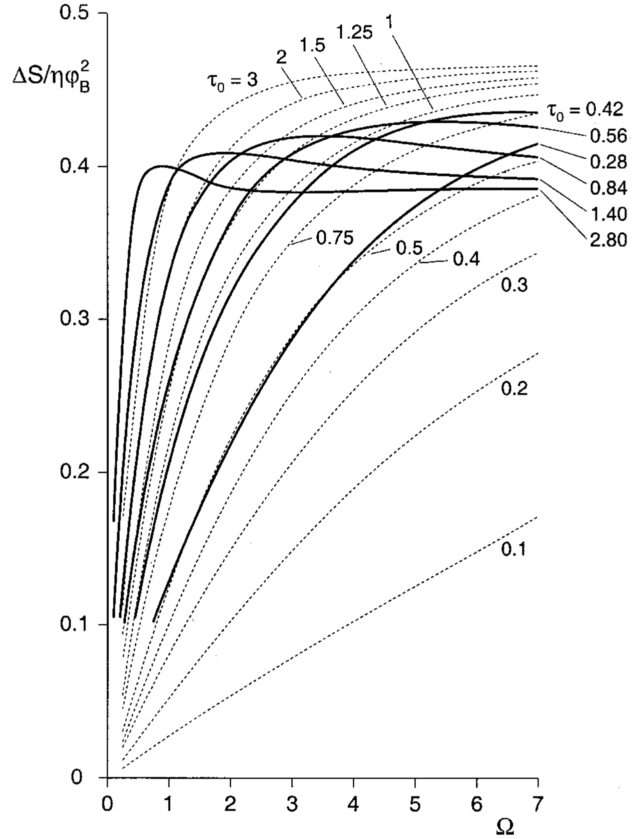

FIG. 3. Increase of the action ΔS in units of $\eta\varphi_B^2$ (continuous lines) as a function of the angular frequency Ω of the bounce trajectory, computed for different values of the delay τ_0 of the artificial line consisting of $N=2-20$ cells, terminated with $Z_T=Z_0$. The dashed curves represent the results of the formula in point (f) of Table I, divided by two. The agreement is acceptable only in the large Ω limit.

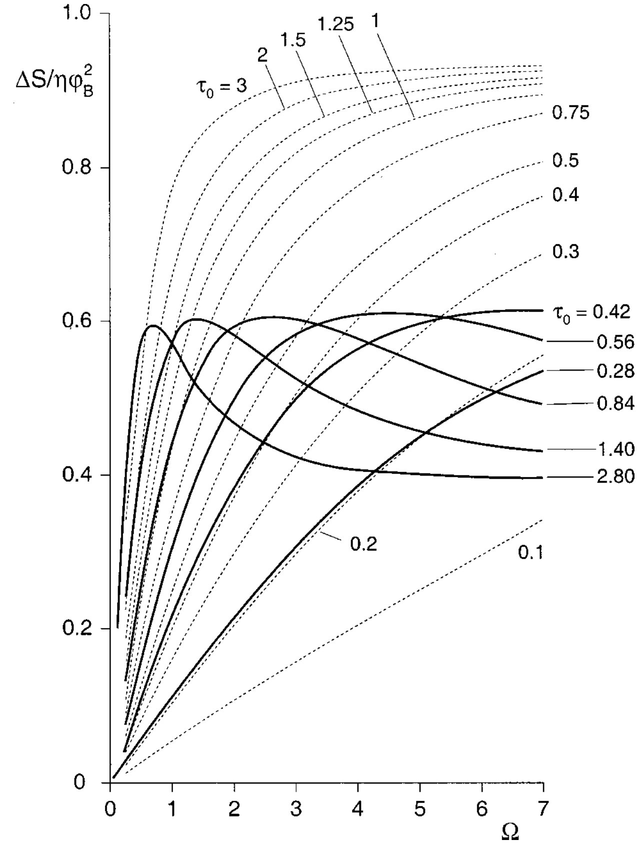

FIG. 4. The same as Fig. 3, but in the case of the same artificial open line ($Z_T=\infty$). The dashed curves represent the results as given by the complete formula in point (f) of Table I. The agreement is acceptable here in the small Ω limit.

of cells, the results for an open line ($Z_T=\infty$) tend to converge on those for a terminated line ($Z_T=Z_0$) [23]. The asymptotic value of ΔS, in units of $\eta\varphi_B^2$, is situated around 0.38, a value slightly less than half of the prediction in point (e) of Table I, or by Eq. (6), that is, 0.436. This confirms the correctness of Eq. (6), which gives an upper limit for numerical evaluations in the case of a terminated line. In this way, all of our results turn out to be situated in a relatively small interval, with an acceptable spreading of values [24]. This confirms the worth of our analysis, which leads, in a rather simple way, to results comparable with the ones obtained by using more complicated and (in some cases) disputable fashions.

III. COMPARISON OF THE THEORETICAL PREDICTIONS WITH THE EXPERIMENTS

Now we wish to test the aforesaid theoretical results against the available experimental results. The latter mainly refer to the determination of the semiclassical traversal time of the barrier, which is obtained by measuring the dependence of the zero-voltage-state lifetime either on the bias current or on the load of the junction [25]. Traversal time results can be used, in turn, for evaluating the dissipative effects as follows.

The semiclassical traversal time of a particle of mass m and energy E, through a potential barrier $V(x)$, is given by [26]

$$\tau=\left(\frac{m}{2}\right)^{1/2}\int_a^b\frac{dx}{[V(x)-E]^{1/2}},\tag{11}$$

where a and b are the turning points at $V(x)=E$. In the presence of dissipative effects, the potential $V(x)$ is augmented by an amount $W(x)=\eta\int_0^x|\dot{x}'|dx'$, which is the equivalent of Eq. (5). The traversal time is thus shortened by an amount given by

$$\Delta\tau=\tau-\tau'=\left(\frac{m}{2}\right)^{1/2}\left(\int_a^b\frac{\mathrm{d}x}{[V(x)-E]^{1/2}}\right.$$
$$\left.-\int_{a'}^{b'}\frac{\mathrm{d}x}{[V(x)+W(x)-E]^{1/2}}\right),\tag{12}$$

where a' and b' are the modified turning points at $V(x)+W(x)=E$. Equation (12) can be evaluated by a perturbative expansion that holds for $W(x)\ll V(x)-E$, and the ratio $\Delta\tau/\tau$ can be expressed as

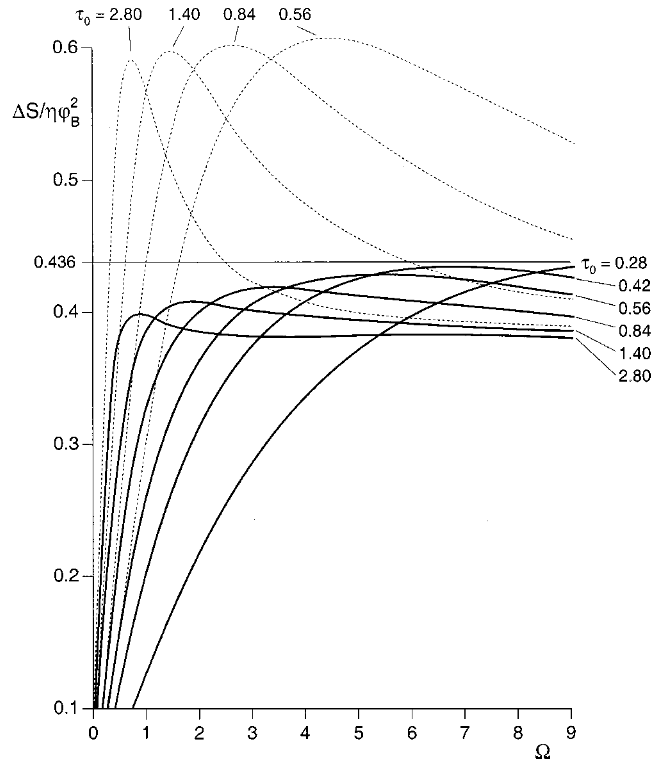

FIG. 5. Comparison of the numerical results of Fig. 3, relative to terminated artificial lines of several lengths (continuous curves), and the results of Fig. 4, relative to open lines (dashed curves). The horizontal line situated at $0.436\eta\varphi_B^2$ corresponds to Eq. (6).

$$\Delta\tau/\tau \simeq \left\langle \frac{W(\tau)}{2[V(x)-E]} \right\rangle, \tag{13}$$

where $\langle\,\rangle$ means an average over the barrier extension. Within the limit of $V_{max}(x) \gg E$, Eq. (13) can be further simplified by taking into account that $W(x) = 2/5\eta\Omega x_B^2 f(x)$, where $f(x)$ is a function whose maximum value at $x=x_B$ holds 2/3 [18], $V_{max} = \Omega S_0/3.6$, where $S_0 = (4/15)m\Omega x_B^2$ is the half-bounce action in the absence of dissipation [5]. We therefore obtain the approximate relation

$$\Delta\tau/\tau \simeq 0.7\frac{\eta x_B^2}{S_0}\overline{f(x)} \approx \frac{2}{3}\,\frac{\Delta S}{S_0} \tag{14}$$

by taking $0.7\overline{f(x)} = 0.31$, where $\Delta S = 0.436\eta\varphi_B^2$, as given by Eq. (6). For our purposes, it is convenient to rewrite Eq. (14) in a different form. Using the substitutions $m \to C_J(\Phi_0/2\pi)^2$ and $\eta \to R^{-1}(\Phi_0/2\pi)^2$—$R$ and C_J being the shunting resistance and capacity of the junction, respectively—the total action becomes $S = S_0 + \Delta S = S_0(1 + 1.64/\Omega RC_J)$.

Therefore, Eq. (14) can be put in the form

$$\frac{\Delta\tau}{\tau} \approx \frac{1.15 \pm 0.25}{Q}, \tag{15}$$

where $Q = \Omega RC_J$, and the uncertainty in the numerical factor roughly accounts for the approximations involved. The results of the numerical analysis, which asymptotically tend to $\sim 0.38\eta\varphi_B^2$ (see Fig. 5), lower the numerical factor in Eq. (15) to ~ 0.95. However, due to the several approximations adopted in deriving Eq. (15), we consider this correction to have less importance. We are now in a position to test the predictions of the theoretical model.

The paper by Voss and Webb [27] allows for a direct test of Eq. (15) since it explicitly supplies the ratio $\Delta S/S_0$ in the form $8A/7.2Q$, where A is a parameter determined by the fitting of the experimental data of the transition rate versus the bias current. The best fit was obtained for $A \simeq 4.5$, which, in Eq. (15), would correspond to a numerical factor of about 3.33, which appears to be a disproportionate value. However, again in Ref. [27], a value of $A \simeq 1.5$ was also considered to be plausible (depending on a different choice of resistance R, hence of coefficient Q). Under this assumption, we obtain a numerical factor of 1.11, which is in excellent agreement with the theoretical prediction.

Another result is offered in the paper by Esteve *et al.* [28]. By assuming their passage time as tunneling time $\tau_t = 78$ ps, to be compared with the half period in harmonic approximation $\tau = \pi/\Omega = 85$ ps, we obtain $\Delta\tau/\tau = 7/85 \simeq 8\%$. Considering that, in this case, $Q = \Omega RC_J = 7.2$ ($\Omega = 3.7 \times 10^{10}$ s^{-1}, $R = 72\,\Omega$, $C_J = 2.7$ pF), in Eq. (15) we obtain a value of 0.575 for the numerical coefficient, which is considerably lower than the prediction of Eq. (15). However, by considering that in the low temperature limit (the result of Ref. [28] refers to a temperature value sufficiently below the crossover temperature) a better determination of the tunneling time is given by $\tau_t = 3.6/\Omega = 97$ ps [25], we obtain $\Delta\tau/\tau = 19/97 = 19.6\%$. This value corresponds to a numerical factor of 1.41, which is in rather good agreement with Eq. (15).

Finally, in Ref. [29] we determined a traversal time of 91 ps for a similar junction with $Q = \Omega RC_J = 11$, where $\Omega = 2 \times 10^{10}$ s^{-1}, $C_J = 6.6$ pF, and $R = 85\,\Omega$, according to Ref. [30]. This time is shorter than the one predicted in harmonic approximation (113 ps), and is preferred in this case since the temperature was not low enough, but rather was comparable with the crossover temperature. Therefore, we have $\Delta\tau = 113 - 91 = 22$ ps and $\Delta\tau/\tau \simeq 19\%$, which, in Eq. (15), corresponds to a numerical factor of 2.1, which is considerably higher than the prediction. A reduction in time of $\sim 19\%$ is presumably a little exaggerated. By considering the curves of the potential barrier for different friction coefficient values [25], we arrive at the conclusion that $\Delta\tau/\tau$ should be around 10%, a value that lowers the numerical factors in Eq. (15) to ~ 1, and is hence in agreement with the test prediction.

We can therefore conclude that, although not rigorous, the criterion expressed by Eq. (15) represents a very practical way of predicting and testing dissipative effects associated with MQT in Josephson junctions. As for the uncertainty in the numerical factor in Eq. (15), the experimental results available are not sufficient enough to select an exact value. Further experimental work needs to be devoted to more precise testing of this numerical factor. However, if more accurate results become available, testing directly on the basis of Eq. (12), rather than by means of the approximate expressions (13) and (15), would be preferable.

[1] A. Yazdani, Nature (London) **409**, 471 (2001).
[2] See, for example, J. Clarke *et al.*, Science **239**, 992 (1988); a more recent account of these arguments can be found in Proceedings of the International Workshop on Macroscopic Quantum Tunneling and Coherence [J. Supercond. **12** (6) (1999)].
[3] K. Fujikawa, S. Iso, M. Sasaki, and H. Suzuki, Phys. Rev. Lett. **68**, 1093 (1992).
[4] A. Widom and T.D. Clark, Phys. Rev. Lett. **48**, 63 (1982); **48**, 1572 (1982); A.O. Caldeira and A.J. Leggett, *ibid.* **48**, 1571 (1982).
[5] For a critical discussion on the problem, see, for example, A. Ranfagni, D. Mugnai, P. Moretti, and M. Cetica, *Trajectories and Rays: The Path-Summation in Quantum Mechanics and Optics* (World Scientific, Singapore, 1990), Vol. 1, Chaps. 7–9.
[6] A.O. Caldeira and A.J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. (N.Y.) **149**, 374 (1983); A.J. Leggett, Phys. Rev. B **30**, 1208 (1984).
[7] L. Chiatti and R. Mignani, Int. J. Mod. Phys. B **11**, 1051 (1997), and references therein.
[8] M. Ueda, Phys. Rev. B **54**, 8676 (1996).
[9] M. Buttiker and R. Landauer, Phys. Rev. Lett. **49**, 1739 (1982).
[10] R. Feynman and A. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965), Chap. 8.
[11] J.P. Sethna, Phys. Rev. B **24**, 698 (1981); **25**, 5050 (1982).
[12] R. Bruinsma and Per Bak, Phys. Rev. Lett. **56**, 420 (1986).
[13] S. Chakravarty and S. Kivelson, Phys. Rev. B **32**, 76 (1985); see also A.T. Dorsey, M.P.A. Fisher, and M.S. Wartak, Phys. Rev. A **33**, 1117 (1986); F. Pignatelli, thesis, University of Florence, 1999 (unpublished).
[14] G. Blatter, Nature (London) **406**, 25 (2000).
[15] A. Ranfagni, D. Mugnai, and R. Englman, Nuovo Cimento D **9**, 1009 (1987).
[16] S. Chakravarty and A. Schmid, Phys. Rev. B **33**, 2000 (1986).
[17] See, for example, A.J. Leggett *et al.*, Rev. Mod. Phys. **59**, 1 (1987).
[18] P. Moretti, D. Mugnai, F. Pignatelli, and A. Ranfagni, Phys. Lett. A **271**, 139 (2000).
[19] For the modifications of the bounce trajectory due to dissipative effects, see P. Moretti, D. Mugnai, A. Ranfagni, and M. Cetica, Phys. Rev. A **60**, 5087 (1999).
[20] K.L. Sebastian and G. Doyen, Phys. Rev. B **47**, 7634 (1993).
[21] A. Widom and T.D. Clark, Phys. Rev. B **30**, 1205 (1984).
[22] An analogous expression is given by S.M. Girvin *et al.*, Phys. Rev. Lett. **64**, 3183 (1990).
[23] The plausibility of this result is well understood considering that for a sufficient length of the line, and for a short pulse duration (high Ω), the impedance seen by the junction is essentially the same, that is, the characteristic impedance of the line $Z_0 = \sqrt{L/C}$.
[24] One might suspect that the numerical analysis does indeed refer to a complete bounce trajectory, rather than to a half bounce. Under this assumption, the result obtained for ΔS should be reduced to one half, that is, ~ 0.19: a value that would make the discrepancy with Eq. (6) much more pronounced, even if still of the right order of magnitude.
[25] P. Fabeni *et al.*, J. Supercond. **12**, 829 (1999).
[26] The definition of traversal time in tunneling is indeed a complicated question, and many different interpretations can be given [see, for example, E.H. Hauge and J.A. Støvneng, Rev. Mod. Phys. **61**, 917 (1989)]. When we refer to the semiclassical traversal time, we mean the one derived in the Wentzel-Kramers-Brillouin nonperturbative semiclassical approximation, as given in Ref. [9]. See also Refs. [19,29].
[27] R.F. Voss and R.A. Webb, Phys. Rev. Lett. **47**, 265 (1981).
[28] D. Esteve *et al.*, Phys. Scr. **T29**, 121 (1989).
[29] A. Ranfagni *et al.*, Phys. Scr. **58**, 538 (1998).
[30] P. Silvestrini *et al.*, Phys. Rev. Lett. **79**, 3046 (1997).